JOURNAL OF PROPULSION AND POWER
Vol. 9, No. 6, Nov.–Dec. 1993

Numerical Studies of Solid Propellant Erosive Burning

Victor K. Bulgakov* and Alexander I. Karpov†
Khabarovsk State University of Technology, Khabarovsk 680035, Russia
and
Alexei M. Lipanov‡
Institute of Applied Mechanics, Izhevsk 426001, Russia

A model of solid propellant erosive burning is developed. Reciprocal interaction of turbulence with chemical reaction is taken into account by considering correlations for turbulent heat transfer affected by combustion reaction in the flame zone and developing an expression for the averaged value of combustion reaction rate. A local-isotropic assumption is invoked to reduce the full transport equations for turbulent fluctuations to simple formulas. Erosive burning of stick propellant is studied numerically by a boundary-layer approach. Satisfactory agreement with measurements is obtained using the proposed model both with the full transport equation and with the local-isotropic assumption. A new physical mechanism of negative erosive burning (decrease of propellant burning rate under blowing of burning surface) is proposed and confirmed by numerical investigations of stick propellant burning by using a mathematical model that is described by two-dimensional Navier-Stokes equations.

Nomenclature

A = pre-exponential factor
a = species mass fraction
B = constant in Eq. (17)
C = specific heat
C_M = constant in Eq. (20)
C_T = constant in Eq. (25)
C_1, C_2, C_3 = constants in Eq. (15)
E = activation energy
f = friction factor
g_t = turbulent heat flux, $-\overline{T'(\rho v)'}$
h = duct's half-thickness
k = kinetic energy of turbulence
Le = Lewis number
l = boundary-layer thickness
m_s = solid propellant mass burning rate
n = reaction's order
Pr = Prandtl number
p = pressure
Q = effective heat of reaction
R = specific gas constant
Re = Reynolds number, $\rho_c u_c h/\mu_m$
R_0 = universal gas constant
T = temperature
t_r = characteristic period of chemical reaction
t_0 = characteristic period of turbulent fluctuations
u = velocity in the x coordinate
Vi = Vilyunov's parameter, $(8\tau_s)^{1/2}/m_s^0$
v = velocity in the y coordinate
W = chemical reaction's rate
x = distance along the burning surface
y = distance normal to the burning surface
α_T, β_T = constants in Eq. (24)
δ = parameter in Eqs. (18) and (19)
ε = dissipation rate of turbulence energy
λ = thermal conductivity
μ = viscosity
ρ = density
τ = shear stress
φ = parameter in Eqs. (17) and (18)

Subscripts
c = core flow
f = flame
g = gas
m = molecular
n = initial
o = input flow
p = solid propellant
r = reaction
s = burning surface
t = turbulent

Superscripts
0 = normal (nonerosive) burning
$\bar{(\ )}$ = Reynolds-averaged variables
$(\ )'$ = fluctuations

Received April 10, 1991; revision received April 17, 1993; accepted for publication May 5, 1993.

*Professor, President of Khabarovsk State University of Technology, Tikhookeanskaya St., 136.
†Associate Professor, Department of Electronic Engineering, Khabarovsk State University of Technology, Tikhookeanskaya St., 136.
‡Professor, Director of Institute of Applied Mechanics, Russian Academy of Sciences, Gorky St., 222.

Introduction

SINCE the effect of erosive burning of solid propellants was discovered, many studies have been devoted to this problem. Concerning the theoretical modeling of this phenomenon, different models were proposed; most of them were reviewed in the survey papers of Kuo and Razdan[1] and King.[2] In addition, some other original works[3–7] may be mentioned. According to King[2] (also Ref. 5), three groups of erosive burning models can be distinguished.

Models of the first group are based on the peculiarities of convective heat transfer from the core flow to the solid propellant burning surface. As King has shown, this mechanism cannot be considered as a fundamental reason of erosive burning because some physical features confirmed by measurements are not handled by this approach. Concerning this mechanism, our numerical investigations[8,9,22] of solid propellants burning under blowing by laminar flow have shown that change of the convective energy balance in the flame zone causes the erosive burning effect (meaning that under this definition any change of burning rate, disregarding physical reasons for this phenomenon). But actually, such an approach

cannot describe one of the main features of erosive burning (confirmed by numerous measurements) that appears as an increase of burning rate with the increase of blowing flow Reynolds number if boundary layer is fully developed. On the other hand, "convective" mechanism was assumed as a basic hypothesis for the theoretical modeling[9,22] of the negative erosive burning (discovered by Vilyunov and Dvoryashin[10,11]), and furthermore, for the study[8] of solid propellant burning under blowing by low Reynolds number flow of complicated configuration.

Returning to the conventional erosive burning process, the last two groups of models (following King's classification) actually deal with the same physical mechanism and differ from each other by different emphases on combustion chemistry vs gas dynamics. This mechanism, first proposed by Corner,[12] attributes erosive burning effects to turbulent heat transfer in the flame zone that leads to increased heat flux into the solid fuel and, correspondingly, to increased burning rate. It should be mentioned that the same approach was proposed by Zeldovich in the 1940s, but accessible publications[13] did not appear until a few decades later. Thus, the problem of erosive burning modeling stands close to the classical problem of turbulent combustion of gases,[14–16] including its peculiarities of theoretical analysis, but complicated by special features like a flow dynamics in the near-wall region and interaction with burning surface of solid fuel. Among other things, two main aspects of these phenomena should be considered: 1) the quest of behaviors by which turbulence affects chemical reaction and 2) the feedback influence of combustion reaction on the turbulent transfer.

In the model of Beddini[4] the generation (or suppression) of turbulent fluctuations provided by chemical reaction is taken into account by using full transport equations for Reynolds stresses, but reaction rate isn't actually averaged, but calculated simply by using averaged parameters in the Arrhenius-type formula. In contrast, Razdan and Kuo[3,17] used the "eddy-break-up" model[18] to describe reaction rate, but did not consider the effect of combustion on turbulent heat transfer. As noted by the authors, this model can be applied only to fully developed turbulent flow. Therefore, characteristics of erosive burning cannot be predicted by this model when Reynolds number isn't sufficiently high for the penetration of the flame zone through the viscous sublayer. In this case molecular transfer should not be neglected in comparison with the turbulent one, and therefore, the eddy-break-up model isn't suitable for predicting reaction rate. Another study of Hsieh et al.[7] is based on using the probability density function to obtain an averaged value of reaction rate by using Favre-averaged equations. This approach seems to be fairly comprehensive, but it is quite difficult to choose a suitable definition of the probability density function.

Summing up the discussed details, two requirements of erosive burning modeling should be addressed. These are 1) accounting for an influence of chemical reaction on the turbulent heat and mass transfer in the flame zone; and 2) developing an expression for an averaged value of reaction rate that would be applicable for any Reynolds number of blowing flow. In addition, a new physical mechanism of negative erosive burning effect is proposed. These subjects are studied in this article, which is a unified analysis of the authors' previous works[8,9,19–24] on this problem.

Model of Erosive Burning Taking into Account an Interaction of Turbulence with Chemical Reaction

To simplify the analysis developed in this article, the following assumptions are imposed:

1) One single-step chemical macroreaction of combustion is taken into account in the gas phase.

2) Following the first assumption, only two species are considered in the gas phase: reactant at mass fraction a and reaction product at mass fraction $(1 - a)$.

The mathematical model of solid propellant erosive burning is described by boundary-layer equations for the flow with chemical reaction. The general form of these equations is

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \mu \frac{\partial u}{\partial y} - \frac{dp}{dx} \tag{1}$$

$$\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \frac{\lambda}{C} \frac{\partial T}{\partial y} + \frac{Q}{C} \rho W \tag{2}$$

$$\rho u \frac{\partial a}{\partial x} + \rho v \frac{\partial a}{\partial y} = \frac{\partial}{\partial y} \frac{Le\lambda}{C} \frac{\partial a}{\partial y} - \rho W \tag{3}$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \tag{4}$$

$$p = R\rho T \tag{5}$$

Reaction rate is described by an Arrhenius-type formula

$$W = A_g a^n \exp\left(-\frac{E_g}{R_0 T}\right) \tag{6}$$

In addition, to avoid superfluous details, Eq. (3) is excluded from the consideration assuming $Le = 1$. In this case, using the similarity between distribution of temperature and mass fraction, the following equation results one:

$$\frac{a}{a_s} = \frac{(T_f - T)}{(T_f - T_s)} \tag{7}$$

The analysis of mass transfer in the case $Le \neq 1$, considered in Ref. 20 is similar to the analysis of heat transfer presented below and doesn't produce any essential new features.

Turbulent Heat Transfer in the Reacting Flow

Applying Reynolds time-averaging procedures to Eqs. (1), (2), (4), and (5) we obtain

$$\bar{\rho}\bar{u} \frac{\partial \bar{u}}{\partial x} + \overline{\rho v} \frac{\partial \bar{u}}{\partial y} = \frac{\partial}{\partial y} \left[\mu_m \frac{\partial \bar{u}}{\partial y} - \overline{u'(\rho v)'}\right] - \frac{\partial \bar{p}}{\partial x} \tag{8}$$

$$\bar{\rho}\bar{u} \frac{\partial \bar{T}}{\partial x} + \overline{\rho v} \frac{\partial \bar{T}}{\partial y} = \frac{\partial}{\partial y} \left[\frac{\lambda_m}{C} \frac{\partial \bar{T}}{\partial y} - \overline{T'(\rho v)'}\right] + \frac{Q}{C} \bar{\rho}\bar{W} \tag{9}$$

$$\frac{\partial \bar{\rho}\bar{u}}{\partial x} + \frac{\partial \overline{\rho v}}{\partial y} = 0 \tag{10}$$

$$\bar{p} = R(\bar{\rho}\bar{T} + \overline{\rho' T'}) \tag{11}$$

Since dynamical turbulence is not affected by chemical reaction in this part of the analysis (uncoupled), the Reynolds stress in the momentum Eq. (8) is described simply as for nonreacting flows:

$$-\overline{u'(\rho v)'} = \mu_t \frac{\partial \bar{u}}{\partial y} \tag{12}$$

Neglecting the fluctuations of pressure, we obtain

$$\overline{\rho' T'} = -(\bar{\rho}/\bar{T})\overline{T'^2} \tag{13}$$

Concerning the parameters of turbulent heat transfer expressed by the correlation $-\overline{T'(\rho v)'}$ in Eq. (9), previous studies[3,7,17] took into account only the conventional expression for nonreacting flow:

$$-\overline{T'(\rho v)'} = \frac{\lambda_t}{C} \frac{\partial \bar{T}}{\partial y} \tag{14}$$

In this analysis the influence of chemical reaction on the turbulent heat transfer is determinated by considering an equation for turbulent heat flux $g_t = -\overline{T'(\rho v)'}$ that can be expressed, following Ref. 25, in the form[23]

$$\bar{\rho}\bar{u}\frac{\partial g_t}{\partial x} + \overline{\rho v}\frac{\partial g_t}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\lambda_m}{C} + C_1\mu_t\right)\frac{\partial g_t}{\partial y} + C_2\bar{\rho}\mu_t\left|\frac{\partial \bar{u}}{\partial y}\frac{\partial \tilde{T}}{\partial y}\right| - C_3\frac{\bar{\rho}\varepsilon}{k_r}g_t + \frac{Q}{C}\bar{\rho}\frac{\partial \tilde{W}}{\partial \tilde{T}}g_t \tag{15}$$

where $C_1 = 1$, $C_2 = 1.6$, $C_3 = 5.4$.

Chemical reaction is postulated to affect turbulent heat transfer by two mechanisms. First, the relaxation process of heat transfer in the flame zone is defined by a new characteristic period t_r, which can be estimated as $t_r \approx 1/\tilde{W}$. Turbulent fluctuations whose periods are greater than t_r cannot affect the distribution of temperature in flame zone because chemical reaction and conduction are faster processes in determination of temperature profiles than turbulence. At the limit of infinitely fast reaction rate ($\tilde{W} \to \infty$, $t_r \to 0$), the temperature gradient becomes infinitely large, the width of reaction zone becomes infinitely thin, and therefore, turbulence doesn't affect heat transfer in the flame zone at all. In the opposite extreme, in the case of very small values of reaction rate, the characteristic period t_r increases and a wider spectrum of turbulent fluctuations affect heat transfer in the flame zone.

Secondly, chemical reaction directly affects the distribution of temperature fluctuations by the last term of Eq. (15). This mechanism was taken into account in the studies of Beddini[4] and Vilyunov and Dik.[26]

Applying the first principles to Eq. (15) we define parameter k_r as a turbulence energy of fluctuations which period is limited by t_r:

$$k_r = k\varphi(t_r/t_0) \tag{16}$$

Here t_0 is the characteristic period of the entire spectrum of turbulent fluctuations.

Defining $t_0 \approx k/\varepsilon$ and using $t_r \approx 1/\tilde{W}$, we find that $t_r/t_0 \approx \varepsilon/k\tilde{W}$. To provide conditions $\varphi(0) \to 0$, $\varphi(1) \to 1$, $\varphi(\infty) = 1$, the correlation function φ is approximated as

$$\varphi = 1 - \exp[-B(\varepsilon/kW)] \tag{17}$$

where B is an empirical constant. As numerical studies have shown $B \approx 1$.

Then Eq. (15) can be rewritten as

$$\bar{\rho}\bar{u}\frac{\partial g_t}{\partial x} + \overline{\rho v}\frac{\partial g_t}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\lambda_m}{C} + C_1\mu_t\right)\frac{\partial g_t}{\partial y} + C_2\bar{\rho}\mu_t\left|\frac{\partial \bar{u}}{\partial y}\frac{\partial \tilde{T}}{\partial y}\right| - C_3\frac{\rho\varepsilon}{k\varphi}g_t(1-\delta) \tag{18}$$

where

$$\delta = \frac{1}{C_3}\frac{k\varphi}{\varepsilon}\frac{Q}{C}\frac{\partial \tilde{W}}{\partial \tilde{T}} \tag{19}$$

Considering assumption of local-isotropic turbulence, Eq. (18) can be significantly simplified. Here, the generally employed expression for turbulent viscosity is used in the form

$$\mu_t = C_M\bar{\rho}k^2/\varepsilon \tag{20}$$

where $C_M = 0.09$.

Keeping only the generation and dissipation terms in Eq. (18), and defining $\mu_t(\partial\bar{u}/\partial y) = -\overline{u'(\rho v)'} \approx 0.3\bar{\rho}k$, Eq. (18) can be transposed to simple formula

$$g_t = \frac{\mu_t}{Pr_t}\frac{\varphi}{(1-\delta)}\frac{\partial \tilde{T}}{\partial y} \tag{21}$$

where $Pr_t = C_3C_M/0.3C_2 \approx 1$.

Equation (21) differs from Eq. (14) for turbulent heat flux in the nonreacting flow by the factor $\varphi/(1-\delta)$. Here, φ describes an influence of chemical reaction on the turbulent heat transfer by the first postulated mechanism, and $(1-\delta)$ by the second one. Therefore, the coefficient of turbulent heat transfer can be expressed from Eqs. (14) and (21) as

$$\lambda_t = \frac{C\mu_t}{Pr_t}\frac{\varphi}{(1-\delta)} \tag{22}$$

The Influence of Turbulent Fluctuations on the Rate of Chemical Reaction

Strong nonlinearity of the dependence of reaction rate upon temperature expressed by Eq. (6) leads to $\overline{W(T)} \neq W(\tilde{T})$. Expanding the function $W(T)$ into a power series and averaging the resulting expression, we obtain

$$\overline{W(T)} = W(\tilde{T}) + 0.5\frac{\partial^2\tilde{W}}{\partial\tilde{T}^2}\overline{T'^2} \tag{23}$$

Considering the energy Eq. (9) and defining

$$-\overline{T'^2(\rho v)'} = \alpha_T\frac{\mu_t}{Pr_t}\frac{\varphi}{(1-\delta)}\frac{\partial\overline{T'^2}}{\partial y}$$

$$\frac{\lambda_m}{C}\frac{\partial\overline{T'}}{\partial y}\frac{\partial\overline{T'}}{\partial y} = \beta_T\frac{\bar{\rho}\varepsilon}{k\varphi}\overline{T'^2}$$

we can derive an equation for $\overline{T'^2}$ in the form

$$\bar{\rho}\bar{u}\frac{\partial\overline{T'^2}}{\partial x} + \overline{\rho v}\frac{\partial\overline{T'^2}}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\lambda_m}{C} + \alpha_T\frac{\mu_t}{Pr_t}\frac{\varphi}{(1-\delta)}\right]\frac{\partial\overline{T'^2}}{\partial y} + 2g_t\frac{\partial\tilde{T}}{\partial y} - 2\frac{\beta_T\bar{\rho}\varepsilon}{k\varphi}(1-\delta)\overline{T'^2} \tag{24}$$

where $\alpha_T = 1$, $\beta_T = 5.4$.

Under the local-isotropic assumption that was applied above to Eq. (18), Eq. (24) can be simplified to

$$\overline{T'^2} = C_T\frac{\mu_t^2}{Pr_t\bar{\rho}^2k}\frac{\varphi^2}{(1-\delta)^2}\left(\frac{\partial\tilde{T}}{\partial y}\right)^2 \tag{25}$$

where $C_T = 1/\beta_TC_M = 2.0$.

Turbulence Model and Boundary Conditions

To complete the set of equations, a turbulence model must be introduced. In the following,[3] the well-known k-ε model is used. Since the viscous sublayer is also considered in the calculations, modification of Chien[27] is used for the prediction of the local characteristics in the flow domain with low Reynolds number.

Finally, the solid propellant burning rate is calculated as[28,38,39]

$$m_s = \left(\frac{\rho_p\lambda_pT_s^2R_0A_p}{E_p[C_p(T_s - T_n) - Q_p/2]}\right)^{1/2}\exp\left(-\frac{E_p}{2R_0T_s}\right) \tag{26}$$

Boundary conditions on the propellant burning surface are described as follows[19]:

$$y = 0: \quad \lambda \frac{\partial T}{\partial y} = m_s(C_g T_s - C_p T_n - Q_p)$$

$$u = 0$$

$$\rho v = m_s$$

Results

Numerical solution of the set of equations presented in the previous sections is obtained by using a finite-difference scheme,[24] which is based on the procedure of Patankar and Spalding[29] for two-dimensional parabolic boundary-layer flows.

Some results of the study of stick propellant erosive burning are presented below. Thermophysical and kinetic properties of these stick propellants and gas-phase parameters are given in Table 1.

The influence of chemical reaction on the turbulent heat transfer in the flame zone is shown in Figs. 1–3. Since the temperature increases monotonically by moving away from a burning surface, according to Eqs. (18) and (19), the chemical reaction generates additional turbulent heat flux g_t in the preflame zone where $\partial\bar{W}/\partial\bar{T} > 0 (\delta > 0)$, and damps g_t on the descending part of function $\bar{W}(\bar{T})$ where $\partial\bar{W}/\partial\bar{T} < 0 (\delta < 0)$. The same result was obtained qualitatively by Vilyunov and Dik.[26] As the results presented in Fig. 1 show, generation of g_t by chemical reaction is almost negligible while suppression is significant. The last conclusion can be seen clearly from the analysis of curve 3 in Fig. 2. The coefficient of turbulent heat transfer whose distribution is shown in Fig. 3 is expressed from Eq. (14) as

$$\lambda_t = C g_t \left(\frac{\partial T}{\partial y}\right)^{-1} \tag{27}$$

Figure 3 shows significant influence of chemical reaction on the turbulent heat transfer in the flame zone. Moreover, in the point $y = 0.15$ mm which corresponds to the point of maximal heat release (see curve 3 in Fig. 1), turbulent heat transfer level is even lower than molecular thermal conductivity. Comparing curves 1 and 2 in Fig. 2, and also, the same curves in Fig. 3, fairly good suitability of the local-isotropic assumption can be concluded.

Before the results of this erosive burning study are considered relative to the prediction of erosivity, one advantage of presented model may be noted. To provide correct prediction of erosivity, normal burning rate (without flow over burning surface) must be calculated using the computational technique that is as close to the technique used for erosive burning as possible. In this regard, the presented model has not assumed

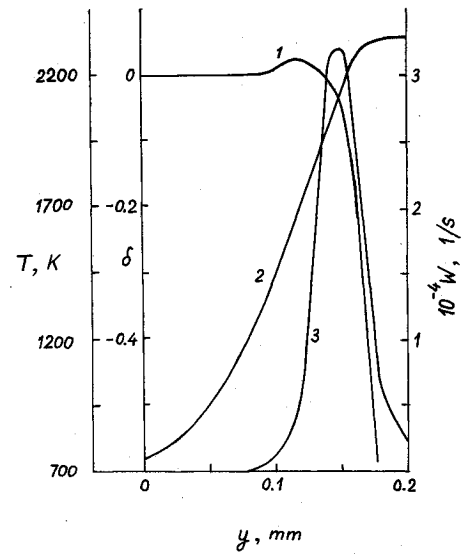

Fig. 1 Distribution of parameter δ (curve 1), temperature (curve 2), and reaction rate (curve 3) in the flame zone.

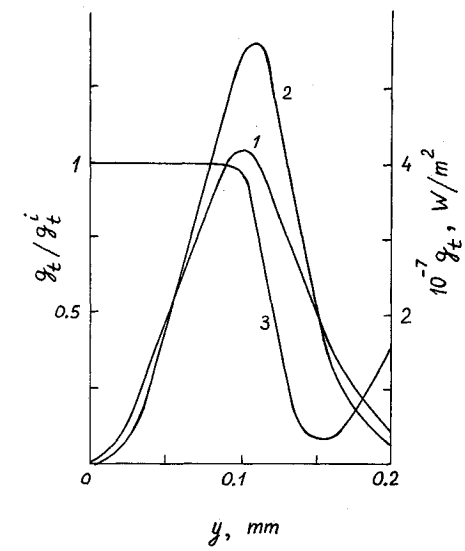

Fig. 2 Influence of chemical reaction on the turbulent heat transfer in the flame zone: curve 1—distribution of turbulent heat flux g_t obtained from the transport Eq. (18); curve 2—distribution of g_t obtained using local-isotropic assumption, Eq. (21); curve 3—distribution of ratio g_t/g_t^i; g_t—turbulent heat flux affected by chemical reaction, obtained from Eq. (18); g_t^i—turbulent heat flux for nonreacting flow, obtained from Eq. (18) under $\delta = 0$, $\varphi = 1$.

Table 1 Solid propellant properties and gas-phase parameters[35–37]

Name	Symbol	Value		Units
		Gas	Solid	
Specific heat	C	1466.5	1465.0	J/(kg·K)
Thermal conductivity	λ	1.0	0.2	W/(m·K)
Density	ρ	14.6[a]	1600.0	kg/m³
Pressure	p	10^7		Pa
Specific gas constant	R	293.0		J/(kg·K)
Universal gas constant	R_0	8.314		J/(mole·K)
Effective heat of reaction	Q	2435.3	556.8	kJ/kg
Pre-exponential factor	A	10^{10}	$3.7 \cdot 10^9$	l/s
Activation energy	E	186.1	79.7	kJ/mole
Initial temperature	T_n		293.0	K
Molecular viscosity	μ_m	$6.2 \cdot 10^{-4}$		kg/(m·s)

[a]At the adiabatic flame temperature $T_f = \frac{C_p}{C_g} T_n + \frac{Q_p}{C_g} + \frac{Q_g}{C_g} = 2333$ K.

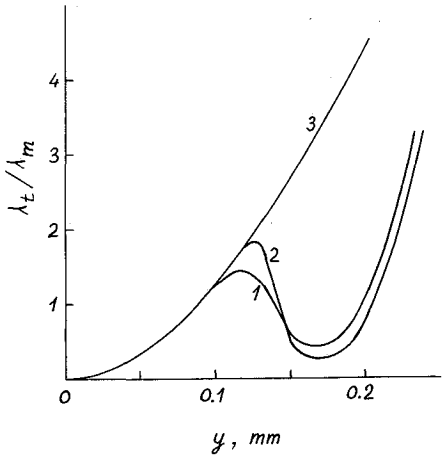

Fig. 3 Distribution of the ratio of turbulent thermal conductivity λ_t to the molecular one λ_m in the flame zone: curve 1—obtained from Eq. (27) using transport equation (18) for g_t; curve 2—obtained by local-isotropic assumption, Eq. (22); curve 3—nonreacting flow, $\delta = 0$, $\varphi = 1$.

any substantial physical distinction between erosive burning and normal burning. Thus, with neglect of turbulent fluctuations and exclusion of convective terms in the x-coordinate direction, the set of Eqs. (9–11) is used to predict the normal burning rate.[24] In this case, the continuity equation (10) is transposed to a simple formula $\rho v = m_s$, and the momentum equation (8) is excluded from the account. To provide numerical solution stability, the unsteady term $\rho(\partial\bar{T}/\partial t)$ is used in Eq. (9) instead of the convective term $\bar{\rho u}(\partial\bar{T}/\partial x)$, with calculations being carried out until a steady value of burning rate is achieved. The model was verified by the predicting of normal burning rate values that agree with measured values.

Measurements of erosive burning of stick propellant over a wide range of blowing velocities of Vilyunov et al.[10,30,31] have shown that erosivity coefficient can be described uniformly as a function of a dimensionless parameter, which was defined in Ref. 19 as Vi. Original expression for this parameter is[32]

$$Vi = \frac{\rho_c u_c}{m_s^0} f^{1/2} \tag{28}$$

Replacing the friction factor with $f = 8\tau_s/\rho_c u_c^2$, Eq. (28) can be rewritten as

$$Vi = \frac{(8\rho_c \tau_s)^{1/2}}{m_s^0} \tag{29}$$

Figure 4 shows substantial disagreement between measurements and predicted results which were obtained neglecting an influence of interaction between chemical reaction and turbulence, while the presented model shows good agreement.

When analyzing results presented in Fig. 4, it can be noticed that negative erosive burning was observed below $Vi = 5.6$. Earlier, some heuristic qualitative description of this phenomenon was considered in Ref. 19. However, direct prediction of negative erosive burning is fairly difficult or even impossible under a boundary-layer approach, because a limiting condition of the existence of a boundary-layer equations' solution depends upon the ratio of stream flow velocity to injection velocity (i.e., burning rate of solid propellant) stands close to the value under which negative erosive burning is observed. For this reason this phenomenon is studied by using a mathematical model described by full two-dimensional Navier-Stokes equations.

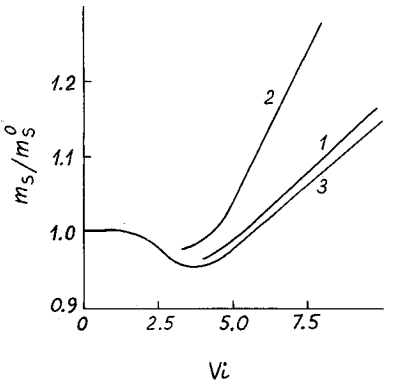

Fig. 4 Erosivity m_s/m_s^0 vs parameter Vi: curve 1—calculation by presented model taking into account the interaction of turbulence with chemical reaction; curve 2—calculation obtained disregarding the interaction of turbulence with chemical reaction; curve 3—measurements.[10]

Negative Erosive Burning

A decrease of solid propellant burning rate under blowing over burning surface, that was defined later as a negative erosive burning, was discovered by Vilyunov and Dvoryashin[10] by measuring burning rates of stick propellants over a wide range of blowing flow Reynolds number, beginning at $Re = 0$. The same authors have also observed negative erosive effects for composite propellants.[11] Negative erosive effects have been obtained under blowing of burning surface by the flow with low Reynolds number or, using parameter Vi as a criterion of erosivity, in the range $1.6 < Vi < 5.6$. First, a systematic error of measurements was considered by some researchers as a reason for negative erosivity appearance. Finally, direct measurement[11] of negative erosive burning confirmed this real physical phenomenon.

The results of numerical investigations[9,22] of the negative erosive burning and the description of a proposed physical mechanism are presented below.

Mathematical modeling of the gas flow is described by two-dimensional Navier-Stokes equations in this study, while the combustion model is the same as that discussed in the previous section. Numerical solution of the two-dimensional equations is obtained by using a finite-difference procedure and the SIMPLE algorithm of Patankar.[33]

The proposed physical mechanism of negative erosive burning is based on the following fundamental premise: negative erosivity doesn't deal with any effect of turbulence. Following this suggestion, burning under laminar flow was studied. According to boundary-layer flow analysis,[34] crossflow velocity appears in the inert duct (without wall injection) over a region where axial velocity profile is not fully developed until boundary layers merge. (In the case of free expanding boundary-layer flow over flat plate, crossflow velocity has a maximal value on the front edge and asymptotically vanishes downstream.) Therefore, if flow over the burning surface is not fully developed, the convective energy balance in the direction normal to burning surface is changed relatively to the non-erosive burning due to additional crossflow convection caused by the boundary layers developing, leading to movement of the flame zone away from the burning surface. As a result, heat flux from the flame zone into the solid propellant decreases, and correspondingly, burning rate drops.

Results of a numerical study of stick propellant burning in the axisymmetric duct formed by two burning surfaces are presented in Figs. 5–8. Propellant properties and gas-phase parameters are the same that are indicated in Table 1. As the results have shown, the parameter Vi is not a universal criterion for prediction of features of the negative erosive effect. An increase of input Reynolds number leads to increased negative erosivity (Fig. 5, curve 1 vs curve 2). An increase of

input boundary-layer thickness leads to decreased negative erosive effect (Fig. 5, curve 1 vs curve 3). In agreement with the suggested theory, the burning rate remains unchanged when input velocity profile is fully developed (Fig. 5, curve 4). Results presented in Fig. 6 give an obvious notion of the proposed mechanism. Negative erosive effects occur until the crossflow velocity in the flame zone exceeds the value that corresponds to nonerosive burning.

Results shown in Figs. 7 and 8 present an erosive burning study in the case of blowing flow that transits from laminar regime to turbulent. Since described physical mechanisms of negative and conventional "positive" erosive effects do not connect with each other, the distribution of curve 1 in Fig. 7 shows that these effects occur in the different regions of the burning surface. According to the previous explanation, burning rate decreases in the developing profile region adjacent

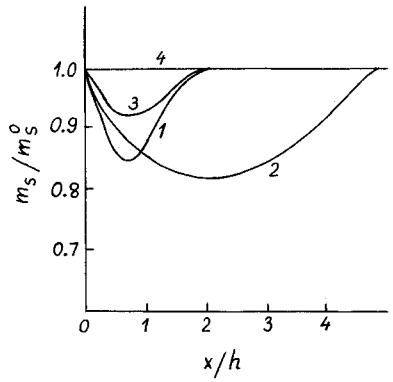

Fig. 5 Distribution of erosivity m_s/m_s^0 over burning surface. Laminar flow. Curve 1—Re_0 = 450, l/h = 0.25; curve 2—Re_0 = 2200, l/h = 0.25; curve 3—Re_0 = 450, l/h = 0.5; curve 4—Re_0 = any, l/h = 1.0.

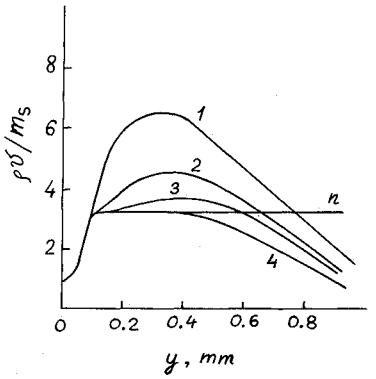

Fig. 6 Distribution of crossflow velocity in the flame zone. Re_0 = 450, l/h = 0.25: curve 1—x/h = 0.55; curve 2—x/h = 1.1; curve 3—x/h = 1.7; curve 4—x/h = 2.7; curve n—nonerosive burning.

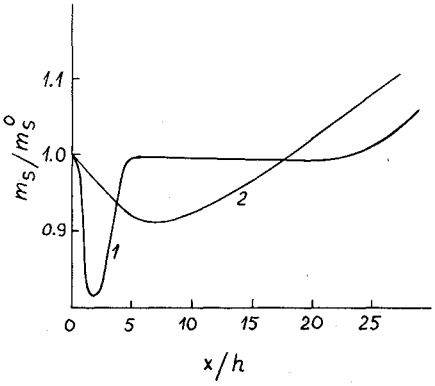

Fig. 7 Distribution of erosivity m_s/m_s^0 over burning surface. Transition flow. Curve 1—Re_0 = 2200, l/h = 0.25; curve 2—Re_0 = 4100, l/h = 0.25.

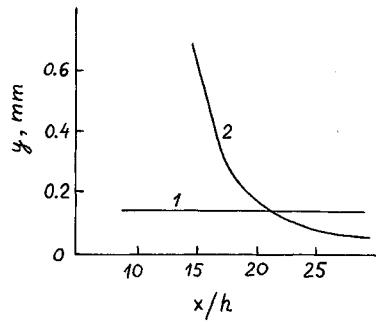

Fig. 8 Location of flame zone (curve 1) and viscous sublayer thickness (curve 2) over burning surface.

to the duct's entrance (actually, parameters of curve 1 in Fig. 7 are the same as of curve 2 in Fig. 5), while an increase of burning rate takes place downstream of the point where transition of laminar flow to turbulent occurs. This last fact is confirmed by the data of Fig. 8 where mutual locations of flame zone and viscous sublayer thickness are shown. Viscous sublayer thickness was determinated as a point where coefficients of turbulent viscosity and molecular viscosity are equal. It can be noticed that the cross section where erosive burning appears (x/h = 22 in Fig. 7) corresponds to the point where the flame zone penetrates into the turbulent core flow (x/h = 22 in Fig. 8). The pattern of curve 2 in Fig. 7 represents typical distribution of erosivity over propellant burning surface under mutual interaction of negative and positive erosive effects.

References

[1]Kuo, K. K., and Razdan, M. K., "Review of Erosive Burning of Solid Propellants," 12th JANNAF Combustion Meeting, CPIA Publication 273, Vol. 11, 1975, pp. 323–338.

[2]King, M. K., "Review of Erosive Burning Models," JANNAF Workshop on Erosive Burning/Velocity Coupling, Lancaster, CA, March 1977.

[3]Razdan, M. K., and Kuo, K. K., "Erosive Burning Study of Composite Solid Propellants by Turbulent Boundary-Layer Approach," *AIAA Journal*, Vol. 17, No. 11, 1979, pp. 1225–1233.

[4]Beddini, R. A., "Reacting Turbulent Boundary-Layer Approach to Solid Propellant Erosive Burning," *AIAA Journal*, Vol. 16, No. 9, 1978, pp. 898–905.

[5]Renie, J. P., and Osborn, J. R., "Erosive Burning," *AIAA Journal*, Vol. 21, No. 12, 1983, pp. 1681–1689.

[6]Lengelle, G., "Model Describing the Erosive Combustion and Velocity Response of Composite Propellants," *AIAA Journal*, Vol. 13, No. 3, 1975, pp. 315–322.

[7]Hsieh, W. H., Char, J. M., Hsieh, K. C., and Kuo, K. K., "Modeling and Measurement of Erosive Burning of Stick Propellant," AIAA Paper 86-1451, July 1986.

[8]Bulgakov, V. K., Karpov, A. I., and Lipanov, A. M., "Effect of Flow Configuration on the Burning Rate of Solid Propellants," *Doklady AN SSSR*, Vol. 312, No. 2, 1990, pp. 391–393.

[9]Bulgakov, V. K., Lipanov, A. M., Vilyunov, V. N., and Karpov, A. I., "On the Mechanism of Negative Erosivity in the Combustion of Solid Fuels," *Fizika Gorenia i Vzryva*, Vol. 25, No. 4, 1989, pp. 32–35.

[10]Vilyunov, V. N., and Dvoryashin, A. A., "An Experimental Investigation of Erosive Burning Effect," *Fizika Gorenia i Vzryva*, Vol. 7, No. 1, 1971, pp. 45–51.

[11]Vilyunov, V. N., and Dvoryashin, A. A., "On the Influence of the Initial Temperature of Solid Fuel on the Effect of Negative Erosivity," *Fizika Gorenia i Vzryva*, Vol. 9, No. 4, 1973, p. 602.

[12]Corner, J., "The Effects of Turbulence on Heterogeneous Reaction Rates," *Transactions of the Faraday Society*, Vol. 43, No. 9, 1947, pp. 635–642.

[13]Zeldovich, Ya. B., "On a Theory of Propellant Burning in a Gas Flow," *Fizika Gorenia i Vzryva*, Vol. 7, No. 4, 1971, pp. 463–476.

[14]Libby, P. A., and Williams, F. A. (ed.), *Turbulent Reacting Flow*, Springer-Verlag, Berlin-Heidelberg, 1980.

[15]Spalding, D. B., "A General Theory of Turbulent Combustion," *Journal of Energy*, Vol. 2, No. 1, 1978, pp. 16–23.

[16]Ghoniem, A. F., Heidarinejad, G., and Krishnan, A., "Tur-

bulence-Combustion Interactions in a Reacting Shear Layer," *Turbulent Reacting Flows: Proceedings of USA-France Joint Workshop*, July 1987, New York, 1989, pp. 638–671.

[17]Razdan, M. K., and Kuo, K. K., "Turbulent Flow Analysis of Erosive Burning of Cylindrical Composite Solid Propellants," *AIAA Journal*, Vol. 20, No. 1, 1982, pp. 122–128.

[18]Spalding, D. B., "Mixing and Chemical Reaction in Steady Confined Turbulent Flames," *13th Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, PA, 1971, pp. 649–657.

[19]Bulgakov, V. K., and Lipanov, A. M., "On Theory of Combustion of Solid Under Blowing," *Fizika Gorenia i Vzryva*, Vol. 19, No. 3, 1983, pp. 32–41.

[20]Bulgakov, V. K., and Lipanov, A. M., "A Model of Solid Fuels Combustion Under Blowing Taking into Account the Interaction of Turbulence with Chemical Reaction," *Fizika Gorenia i Vzryva*, Vol. 20, No. 5, 1984, pp. 68–74.

[21]Bulgakov, V. K., Lipanov, A. M., and Kamaletdinov, A. S., "Numerical Study of Erosive Burning of Solids," *Fizika Gorenia i Vzryva*, Vol. 22, No. 6, 1986, pp. 83–88.

[22]Bulgakov, V. K., and Karpov, A. I., "Numerical Investigation of the Flow Feature Influence on the Combustion of Solids," *Third International Symposium on Computational Fluid Dynamics*, Nagoya Univ., Nagoya, Japan, 1989, pp. 903–906.

[23]Bulgakov, V. K., and Lipanov, A. M., "The Interaction of Turbulence with Chemical Reaction in the Theory of Erosive Burning of Solids," *Khimicheskaya Fizika*, Vol. 5, No. 4, 1986, pp. 548–556.

[24]Bulgakov, V. K., Lipanov, A. M., and Kamaletdinov, A. S., "The Calculation of Burning Rate of Solid Propellants Under Blowing," *Khimicheskaya Fizika*, Vol. 5, No. 6, 1986, pp. 831–837.

[25]Reynolds, A. J., *Turbulent Flows in Engineering*, Wiley, New York, 1974.

[26]Vilyunov, V. N., and Dik, I. G., "On the Statistical-Phenomenological Approach in the Analysis of Turbulent Flames," *Zhurnal Prikladnoi-Mekhaniki i Tekhnicheskoi Fiziki*, No. 5, 1976, pp. 61–68.

[27]Chien, K. Y., "Prediction of Channel and Boundary Layer Flows with Low Reynolds Number Turbulence Model," *AIAA Journal*, Vol. 20, No. 1, 1982, pp. 33–38.

[28]Lengelle, G., "Thermal Degradation Kinetics and Surface Pyrolysis of Vinyl Polymers," *AIAA Journal*, Vol. 8, No. 11, 1970, pp. 1984–1996.

[29]Patankar, S. V., and Spalding, D. B., *Heat and Mass Transfer in Boundary Layers*, Intertext Books, London, 1970.

[30]Vilyunov, V. N., Dvoryashin, A. A., Margolin, A. D., Ordzhonikidze, S. K., and Pokhil, P. F., "The Combustion of Stick Propellant 'N' in the Sonic Flow," *Fizika Gorenia i Vzryva*, Vol. 8, No. 4, 1972, pp. 501–505.

[31]Vilynov, V. N., Isaev, Y. M., and Kuznetsov, A. T., "Study of Turbulence Structure Influence on Erosive Combustion of Solid Propellants," *Fizika Gorenia i Vzryva*, Vol. 17, No. 3, 1981, pp. 133–135.

[32]Vilyunov, V. N., "On Theory of the Erosive Burning of Powders," *Doklady AN SSSR*, Vol. 136, No. 2, 1961, pp. 381–383.

[33]Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, Hemisphere, Washington, DC, 1980.

[34]Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, New York, 1968.

[35]Zenin, A. A., "A Formal Kinetic Characteristics of the Reactions of Propellant Burning," *Fizika Gorenia i Vzryva*, Vol. 2, No. 2, 1966, pp. 28–38.

[36]Zenin, A. A., "The Structure of the Temperature Distribution Under the Steady Burning of Stick Propellants," *Fizika Gorenia i Vzryva*, Vol. 2, No. 3, 1966, pp. 67–76.

[37]Zenin, A. A., "Processes in the Burning Zones of Stick Propellants," *Physical Processes of Combustion and Explosion*, Atomizdat, Moscow, 1980, pp. 68–105.

[38]Merzhanov, A. G., and Dubovitskiy, F. I., "On the Theory of Stationary Burning of Powder," *Doklady AN SSSR*, Vol. 129, No. 1, 1959, pp. 153–158.

[39]Novozhilov, B. V., *Nonstationary Burning of Solid Rocket Propellants*, Nauka, Moscow, 1973.